

Hk 2, 105 Written problems

The simplest written problems just say something about some numbers, and ask what the numbers are. First we need to be able to translate words into algebraic expressions. For each phrase below write an algebraic expression. Use x for the unknown number.

Example: Write an algebraic expression for two less than four times a number

Answer: $4x - 2$

Example: add five to a number, then divide the result by six

Answer: $\frac{x + 5}{6}$

Note that these answers are just expressions, not equations. There is nothing to "solve" yet.

Write an expression for:

1. the sum of twice a number and 7
2. four times a number taken away from sixteen
3. five times the result you get when you subtract four from a number
4. seven less than three times a number
5. the sum of three times a number and the negative of the number

6. the result you get when you divide fifteen by four times a number

The following problems can be translated to equations. First declare the variable, then write the equation, then solve, then make sure you have answered the question in the problem.

Example: If you take 14 away from five times a number you get 61. Find the number.

$$\begin{array}{ll} \text{let } x = \text{the number} & \text{(declare variable)} \\ 5x - 14 = 61 & \text{(equation)} \\ 5x = 75 & \\ x = 15 & \text{(solution)} \end{array}$$

Check: Is it true that if you take 14 away from five times 15 you will get 61?

Example. There are two numbers that add up to 29. The smaller number is one less than twice the larger. Find the two numbers.

$$\begin{array}{l} \text{let } x = \text{larger number} \\ \text{then } 2x - 1 = \text{smaller number} \\ x + (2x - 1) = 29 \quad \text{solve for } x \end{array}$$

In each written problem declare the variable, set up and equation, and solve:

7. Nine more than twelve times a number is equal to 93. Find the number

8. Five times a number is equal to the sum of twice the number and seven. Find the number.

10. Find three consecutive integers whose sum is forty two.

9. There are two numbers. The smaller one is two more than one half the larger one. When you add them you get twenty-six. Find both numbers.

11. Find three consecutive odd integers whose sum is 93. (hint: If x is an odd integer, the next odd integer is not $x + 1$. What is it?)

12. Find three consecutive even integers whose sum is 93. (hint: Think about this.)

Example: Find three consecutive integers whose sum is 69.

Let x = the first integer

then $x + 1$ = second integer

and $x + 2$ = the third integer

$$x + (x + 1) + (x + 2) = 69$$

$$3x + 3 = 69$$

$$3x = 66$$

$$x = 22, \text{ first integer}$$

$$x + 1 = 23, \text{ second integer}$$

$$x + 2 = 24, \text{ third integer}$$

Hk 3, Math 105 Coin problems

Example 1: A pile of coins consists of dimes and quarters. There are three more dimes than quarters. The total value of the coins is \$4.15. How many dimes are there, and how many quarters?

First declare your variables:

$$\begin{aligned}x &= \text{number of quarters} \\x + 3 &= \text{number of dimes}\end{aligned}$$

Next comes a very important step. The value of coins is different than the number of coins. If you have x quarters and each quarter is worth $.25$, then you multiply to get the total value of the quarters. Then figure the value of the dimes.

$$\begin{aligned}.25x &= \text{value of the quarters} \\ .1(x + 3) &= \text{value of the dimes}\end{aligned}$$

You could state the value of the quarters as $25x$, and the value of the dimes as $10(x + 3)$, but then you are working in pennies, not dollars. Usually we would expect to work in dollars. The total value is given in dollars, not pennies. Now you can make an equation. The problems says the total value of all the coins is \$4.15.

$$.25x + .1(x + 3) = 4.15$$

The left side of your equation is a value. The right side of your equation is a value. They are equal, but only if you find the right value of x . Solve the equation to find that value.

$$\begin{aligned}.25x + .1(x + 3) &= 4.15 \\ .25x + .1x + .3 &= 4.15 \\ .35x + .3 &= 4.15 \\ .35x &= 3.85 \\ x &= 11 \text{ quarters} \\ x + 3 &= 14 \text{ dimes}\end{aligned}$$

Example 2: Eighteen coins have a total value of \$3.10. Some of the coins are nickels and some are quarters. How many of each kind are there?

$$\begin{aligned}\text{let } x &= \text{number of nickels.} \\ \text{then } 18 - x &= \text{number of quarters} \\ .05x &= \text{value of the nickels} \\ .25(18 - x) &= \text{value of the quarters.}\end{aligned}$$

$$.05x + .25(18 - x) = 3.10$$

Solve for x . Make sure you answer all questions in the problem.

Problems - Make sure you declare variables and answer what is asked.

1. Thirty-five coins have a total value of \$2.75. Some of the coins are nickels and some are quarters. How many of each are there?

2. The total value of nickels and dimes is \$8.00. The number of nickels is equal to twice the number of dimes. Find the number of each.

3. The number of nickels is six more than five times the number of dimes. The total value of these coins is \$2.40. How many nickels and how many dimes are there.

5. In an envelope there are some 42 cent stamps and some 3 cent stamps. The number of 42 cent stamps is two less than the number of 3 cent stamps. The total value of all the stamps is \$5.01. Find the number of each type of stamp.

4. In a jar full of coins there are 14 more dimes than three times the number of nickels, and there are 62 coins in all. Find the number of each type of coin. (Hint: This problem is different in a very important way from the previous problems. Think on it!)

Distance problems: Distance equals rate times time. If the rate is 65 miles per hour and you travel for three hours, you find the distance traveled by multiplying rate times time.

$$195 \text{ miles} = 65 \text{ mph} \times 3 \text{ hr}$$

This gives the equation:

$$d = rt$$

This equation can be solved for rate, or it can be solved for time.

$$r = d/t$$

$$t = d/r$$

So if you know any two of the three quantities, distance, rate, and time, you can write an expression for the third.

Practice: Fill in the blanks in the chart.

	distance	rate	time
1		55 mph	2 hours
2	300 miles		4.5 hrs
3	720 miles	220 mph	
4		x	3
5	X + 200		6
6		2X - 50	45

7. Fred and Joe start 270 miles apart and drive directly toward each other until they meet. Fred travels at 40 mph and Joe travels at 50 mph. In how many hours will they meet? How far will each travel? Start by filling in the chart. Let x be the time for Fred. x will also be the time for Joe. Then fill in the rate box. The distance box will be an algebraic expression you get by multiplying rate times time. The equation will say that Joe's distance plus Fred's distance equals 270 miles. Solve the equation. For full credit on these problems you must show a chart filled in fully and correctly with expressions of the variable, and the equation must be stated correctly and solved

correctly, and all questions answered.

	distance	rate	time
Fred			
Joe			

8. Two airplanes start from the same airport and travel directly away from each other. Plane A travels at 240 mph. Plane B starts one hour later than plane A and travels at 300 mph. When will they be 1050 miles apart? How far will each plane travel?

(This problem is very similar to 7, but not identical. Start with a chart. Let x = time for plane A. Find an expression for the time of plane B. Find expressions for the distance each plane travels. Write an equation that says the sum of the two distances is 1050 miles.)

9. Alice and Burt have a fight. Alice gets mad and leaves, traveling south at 45 miles per hour. One half hour later Burt goes after her, traveling at 56 miles per hour. How long will Burt travel to catch up with Alice?

10. A bicyclist takes two hours to ride up a mountain. He returns by the same route in only one and a half hours, because his speed is six miles per hour faster. Find his speed coming and going. How far is his route one way?

Hk 5, Math 105, Mixture problems

Example: How many pounds of beans worth \$1.20 a pound must be mixed with 30 pounds of beans worth \$1.50 a pound to make a mixture worth \$1.40 a pound.

	price/lb	quantity	value
cheap	1.2	x	1.2x
costly	1.5	30	45
mixture	1.4	30 + x	

That last box can be filled in two ways. You can multiply across the third row to get $1.4(30 + x)$. Or you can add down the third column to get $1.2x + 45$. So there is your equation:

$$1.4(30 + x) = 1.2x + 45 \quad \text{solve for } x$$

The problem does not ask how many pounds of beans there will be altogether, but if you want to check out your answer you will need to know that. The problem does not ask for the total value of all the beans, but again if you want to check it out you will have to know that.

Ex. How many pounds of candy worth \$2.50 a pound must be mixed with candy worth \$3.10 a pound to make 20 pounds of candy worth \$2.74 a pound?

	price/lb	quantity	value
cheap	2.5	x	2.5x
costly	3.10	20 - x	3.1(20-x)
mixture	2.74	20	

Again that last column can be filled in two ways. You can multiply across the third row to get 5.6, or you can add down the third column to get $2.5x + 3.1(20 - x)$. So that is the equation:

$$2.5x + 3.1(20 - x) = 2.74(20) \quad \text{solve for } x$$

And again to check your answer to know that it is really right you will have to find some amounts not asked for in the problem.

Problems: (Declare your variables with a chart, set up a equation. In other words, use algebra. Definitely use a calculator when needed.)

1. How many pounds of nuts worth \$4.50 a pound must be added to 600 pounds of nuts worth \$5.40 a pound to get a mixture worth \$4.80 a pound?

2. How many pounds of red plastic beads worth \$.30 a pound must be mixed with clear plastic beads worth \$.80 a pound to make 60 pounds of mixed beads worth \$.58 a pound?

3. How many liters of 20% orange juice concentrate must be mixed with 700 liters of 38% orange juice concentrate to make a mixture that is a 35% concentrate?

5. A sum of \$8000 is invested in bonds, part at 8% and part at 14%. The total return on the investment is \$840. How much is invested at each rate? (Hint: This is very similar to a mixture problem. Start by filling in the chart.)

	rate	amount	return
first part			
other part			

4. How many liters of a 40% alcohol solution can be mixed with how many liters of a 48% alcohol solution to make 800 liters of a 42% alcohol solution?